National
Qualifications
2023 MODIFIED

# X847/76/11

# Mathematics
# Paper 1 (Non-calculator)

THURSDAY, 4 MAY

9:00 AM – 10:15 AM

**Total marks — 55**

Attempt ALL questions.

**You must NOT use a calculator.**

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.



*X8477611*

A/HTP

## FORMULAE LIST

### Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

### Scalar product

$\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

### Trigonometric formulae

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin\mathrm{A}\cos\mathrm{B} \pm \cos\mathrm{A}\sin\mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos\mathrm{A}\cos\mathrm{B} \mp \sin\mathrm{A}\sin\mathrm{B}$$

$$\sin 2\mathrm{A} = 2\sin\mathrm{A}\cos\mathrm{A}$$

$$\begin{aligned} \cos 2\mathrm{A} &= \cos^2\mathrm{A} - \sin^2\mathrm{A} \\ &= 2\cos^2\mathrm{A} - 1 \\ &= 1 - 2\sin^2\mathrm{A} \end{aligned}$$

### Table of standard derivatives

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a\cos ax$ |
| $\cos ax$ | $-a\sin ax$ |

### Table of standard integrals

| $f(x)$ | $\int f(x)dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a}\cos ax + c$ |
| $\cos ax$ | $\frac{1}{a}\sin ax + c$ |

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**Attempt ALL questions**

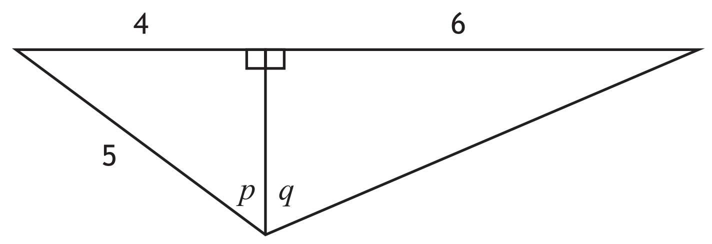
**1.** Given that $y = x^{\frac{5}{3}} - \frac{10}{x^4}$, where $x \neq 0$, find $\frac{dy}{dx}$. **3**

**2.** P and Q are the points $(-2, 6)$ and $(10, 0)$.

Find the equation of the perpendicular bisector of PQ. **4**

**3.** Solve $\log_5 x - \log_5 3 = 2$. **3**

**4.** The diagram shows two right-angled triangles with angles $p$ and $q$ as marked.




(a) Determine the value of:

(i) $\cos p$ **1**

(ii) $\cos q$. **1**

(b) Hence determine the value of $\cos(p + q)$. **3**

MARKS

**5.** The equation $2x^2 + (3p-2)x + p = 0$ has equal roots.

Determine the possible values of $p$. **3**

**6.** Find $\int \left(2x^5 - 6\sqrt{x}\right) dx,\ x \geq 0$. **4**

**7.** (a) Evaluate $\log_2 5 + \log_2 \frac{1}{40}$. **2**

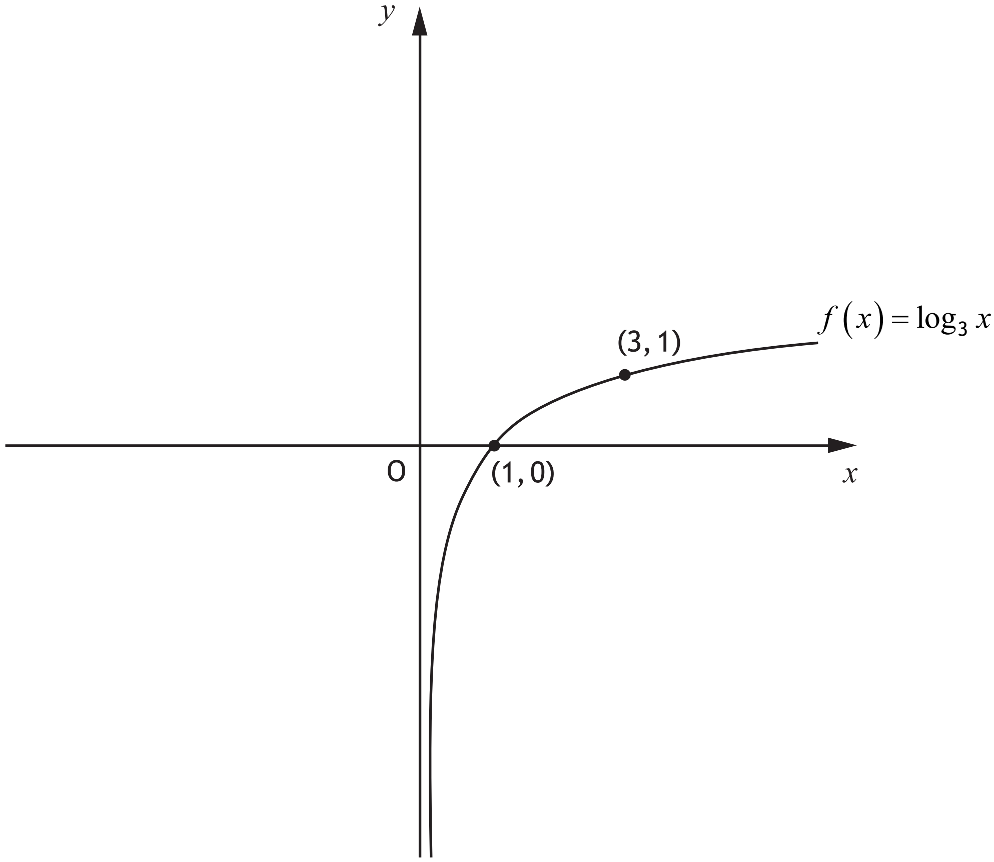
(b) Given that $a \in \mathbb{R}$ and that $\log_8 a$ is negative, state the range of possible values of $a$. **1**

**8.** A function, $f$, is defined on $\mathbb{R}$, the set of real numbers, by $f(x) = x^3 + 3x^2 - 9x + 5$.

Find the coordinates of the stationary points of $f$ and determine their nature. **6**

MARKS

**9.** The diagram shows the graph of the function $f(x) = \log_3 x$, where $x > 0$.




The inverse function, $f^{-1}$, exists.

On the diagram in your answer booklet, sketch the graph of $y = f^{-1}(x) - 1$. **3**

**10.** (a) Show that $(x+5)$ is a factor of $x^4 + 3x^3 - 7x^2 + 9x - 30$. **2**

(b) Hence, or otherwise, solve $x^4 + 3x^3 - 7x^2 + 9x - 30 = 0$, $x \in \mathbb{R}$. **5**

**[Turn over**

MARKS

**11.** (a) Evaluate $\int_{\frac{\pi}{2}}^{\pi}\left(5\sin x-3\cos x\right)\,dx$. **3**

The diagram in your answer booklet shows the graphs with equations $y=5\sin x$ and $y=3\cos x$ , $0\leq x\leq 2\pi$.

(b) On the diagram in your answer booklet, shade the area represented by the integral in (a). **1**

**12.** Express $-2x^2-12x+7$ in the form $a\left(x+b\right)^2+c$. **3**

**13.** Functions $f$ and $g$ are defined by:

- $f\left(x\right)=2\sin x$, where $0<x<\frac{\pi}{2}$
- $g\left(x\right)=2x$, where $0<x<\frac{\pi}{4}$

(a) (i) Evaluate $f\left(g\left(\frac{\pi}{6}\right)\right)$. **1**

(ii) Determine an expression for $f\left(g\left(x\right)\right)$. **2**

(b) (i) Given that $f\left(p\right)=\frac{1}{3}$, determine the exact value of $\sin p$. **1**

(ii) Hence, determine the exact value of $f\left(g\left(p\right)\right)$. **3**

**[END OF QUESTION PAPER]**

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# X847/76/12

# Mathematics
# Paper 2

THURSDAY, 4 MAY

10:45 AM – 12:15 PM

**Total marks — 65**

Attempt ALL questions.

**You may use a calculator.**

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*X8477612*

A/HTP

## FORMULAE LIST

### Circle

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The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Scalar product** $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

**Trigonometric formulae**

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B}$$

$$\sin 2\mathrm{A} = 2 \sin \mathrm{A} \cos \mathrm{A}$$

$$\begin{aligned} \cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\ &= 2\cos^2 \mathrm{A} - 1 \\ &= 1 - 2\sin^2 \mathrm{A} \end{aligned}$$

**Table of standard derivatives**

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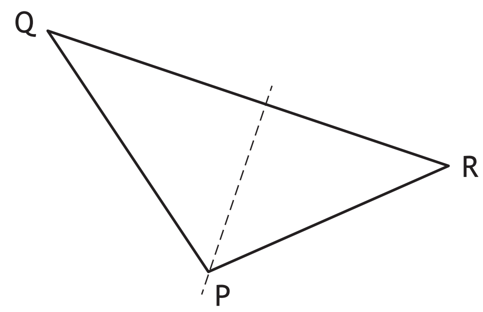
MARKS

**Total marks — 65**

**Attempt ALL questions**

**1.** Triangle PQR has vertices P(5, –1), Q(–2, 8) and R(13, 3).




(a) Find the equation of the altitude from P. **3**

(b) Calculate the angle that the side PR makes with the positive direction of the $x$-axis. **2**

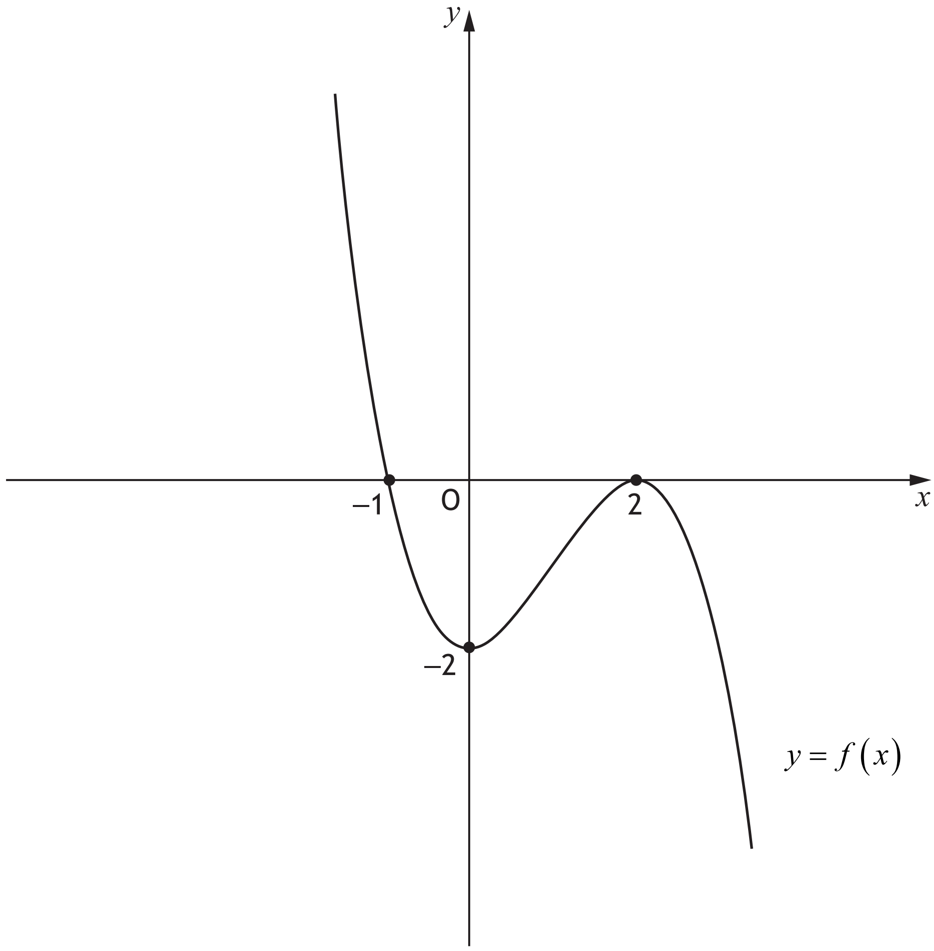
**2.** Find the equation of the tangent to the curve with equation $y = 2x^5 \quad 3x$ at the point where $x = 1$. **4**

**3.** Find $\int 7\cos\left(4x + \frac{\pi}{3}\right)dx$. **2**

**[Turn over**

**MARKS**

**4.** The diagram shows the cubic graph of $y = f(x)$, with stationary points at (2, 0) and (0, –2).




On the diagram in your answer booklet, sketch the graph of $y = 2f(\quad x)$. **2**

**5.** A function, $f$, is defined by $f(x) = (3 \quad 2x)^4$, where $x \in \mathbb{R}$.

Calculate the rate of change of $f$ when $x = 4$. **3**

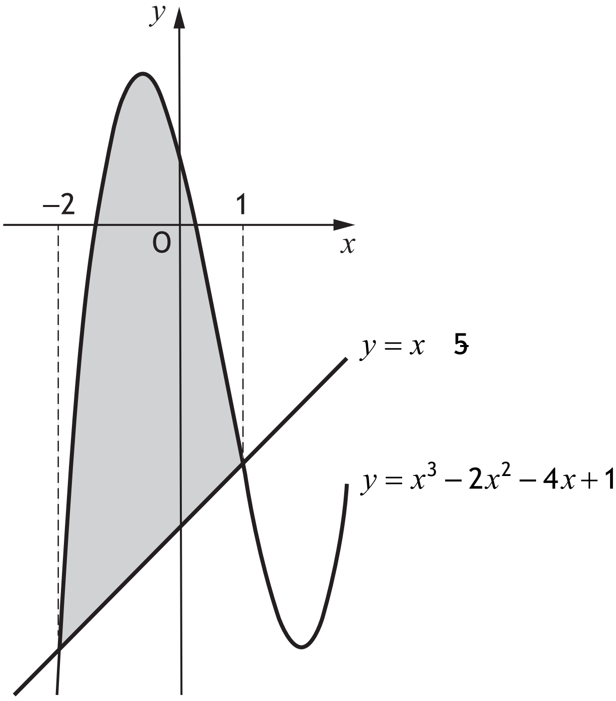
**6.** A function $f(x)$ is defined by $f(x) = \frac{2}{x} - 3$, $x > 0$.

Find the inverse function, $f^{-1}(x)$. **3**

**7.** Solve the equation $\sin x° + 2 = 3\cos 2x°$ for $0 \leq x < 360$. **5**

**8.** The diagram shows part of the curve with equation $y = x^3 - 2x^2 - 4x + 1$ and the line with equation $y = x - 5$.

The curve and the line intersect at the points where $x = -2$ and $x = 1$.




Calculate the shaded area. **5**

**[Turn over**

MARKS

**9.** (a) Express $7\cos x° - 3\sin x°$ in the form $k\sin(x+a)°$ where $k > 0$, $0 < a < 360$. **4**

(b) Hence, or otherwise, find:

(i) the maximum value of $14\cos x° - 6\sin x°$ **1**

(ii) the value of $x$ for which it occurs where $0 \le x < 360$. **2**

**10.** Determine the range of values of $x$ for which the function $f(x) = 2x^3 + 9x^2 - 24x + 6$ is strictly decreasing. **4**

**11.** Circle $C_1$ has equation $(x-4)^2 + (y+2)^2 = 37$.
Circle $C_2$ has equation $x^2 + y^2 + 2x - 6y - 7 = 0$.

(a) Calculate the distance between the centres of $C_1$ and $C_2$. **3**

(b) Hence, show that $C_1$ and $C_2$ intersect at two distinct points. **3**

MARKS

**12.** A curve, for which $\frac{dy}{dx} = 8x^3 \quad 3$, passes through the point $(-1, 3)$.

Express $y$ in terms of $x$. **4**

**13.** A patient is given a dose of medicine.

The concentration of the medicine in the patient's blood is modelled by

$$C_t = 11e^{-0.0053\,t}$$

where:

- $t$ is the time, in minutes, since the dose of medicine was given
- $C_t$ is the concentration of the medicine, in mg/l, at time $t$.

(a) Calculate the concentration of the medicine 30 minutes after the dose was given. **1**

The dose of medicine becomes ineffective when its concentration falls to 0.66 mg/l.

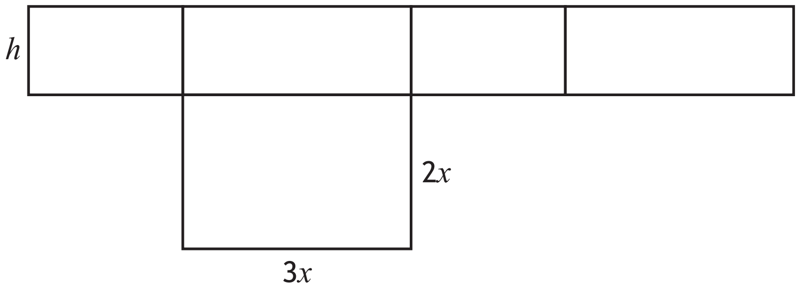
(b) Calculate the time taken for this dose of the medicine to become ineffective. **3**

**[Turn over**

**14.** A net of an open box is shown.

The box is a cuboid with height $h$ centimetres.

The base is a rectangle measuring $3x$ centimetres by $2x$ centimetres.

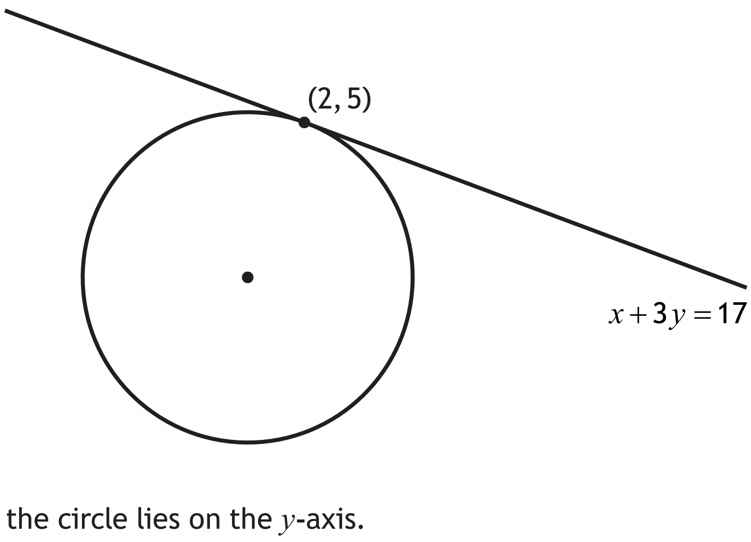



(a) (i) Express the area of the net, $A$ cm$^2$, in terms of $h$ and $x$. **1**

(ii) Given that $A = 7200$ cm$^2$, show that the volume of the box, $V$ cm$^3$, is given by $V = 4320x - \frac{18}{5}x^3$. **2**

(b) Determine the value of $x$ that maximises the volume of the box. **4**

**15.** The line $x + 3y = 17$ is a tangent to a circle at the point $(2, 5)$.




The centre of the circle lies on the $y$-axis.

Find the coordinates of the centre of the circle. **4**

**[END OF QUESTION PAPER]**